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Book Review

Publishers are invited to send two copies of new books for review to Dr. I. Michael Ross, Code: AA/Ro, Department of Aeronautics and Astronautics, U.S. Naval Postgraduate School, 699 Dyer Road, Monterey, CA 93943.

Tensor Calculus and Analytical Dynamics: A Classical Introduction to Holonomic and Nonholonomic Tensor Calculus and Its Principal Applications to the Lagrangean Dynamics of Constrained Mechanical Systems. For Engineers, Physicists, and Mathematicians

John G. Papastavridis, CRC Press, 1998, xvii + 409 pp., $69.95.

"What! Another book on vectors and tensors?... What conceivable reason can this fellow have for inflicting another book on us?" Thus began the preface of a 1962 text,[1] whose author (R. Aris) went on to justify this infliction to his intended audience of engineering scientists, physicists, and applied mathematicians. He observed that the engineer's "knowledge of mathematics must go beyond a nodding acquaintance with its notions and notations," and that "tensor calculus is the natural language of continuum or field theories...." These propositions set the stage for a thoughtful, balanced presentation of tensor calculus that is still a valuable addition to the library of any mathematically inclined engineer.

Aris's notional question and subsequent reasoning are as valid today as they were nearly 40 years ago. We might well ask the same question of Papastavridis, whose intended audience (judging from the monograph's subtitle) overlaps substantially with Aris's. How would Papastavridis answer? Turning to the Preface, we learn that his book is "for people who place theories, ideas and knowledge ... above all else—irrespectively of their age, or short-term profitability, expediency, or the *ad nauseum* parroted 'computational efficiency'—and do not apologize for it." This confrontational opening sets the stage for a pedantic presentation of tensor calculus and its application to Lagrangean mechanics, sprinkled throughout with colorful, though perhaps biased, accounts of contemporary treatments of the subject.

The book is divided into two Parts: Tensor Calculus, and Analytical Dynamics. Part I: Tensor Calculus comprises three chapters: Introduction and Background, Tensor Algebra, and Tensor Analysis. Part II: Analytical Dynamics comprises four chapters: Introduction to Analytical Dynamics, Particle on a Curve and on a Surface, Lagrangean Mechanics: Kinematics, and Lagrangean Mechanics: Kinetics. The text includes many useful results from both tensor calculus and analytical dynamics, including many results on nonholonomic systems. Also included are many worked examples and exercises. Of particular interest to readers of this journal are the examples analyzing the kinematics and kinetics of particles and rigid bodies. While these examples are at a basic level, involving only a single particle or a single rigid body, they serve to illustrate the tensor analysis techniques covered in the book. The careful student can readily extend these examples to cover the more complicated problems encountered in vehicle dynamics applications.

A six-page index provides adequate pointers to the material in the book. Curiously absent from this list is any instance of the various forms of the word "Hamilton" that are sometimes found in texts on analytical dynamics. Hamilton is mentioned, however, on p. 169 in a list of 16 ("*et al*") researchers who "elaborated and perfected" Lagrangean dynamics. Some 200 references are included in the bibliography, many of which are classic treatises on the subject. However, several of the references that are discussed in the text are missing from the bibliography. For example, on p. 308, the Routh–Voss equations are introduced, citing Routh's 1877 treatise and Voss's 1885 study, neither of which are included in the bibliography. This is an unfortunate oversight in an otherwise carefully prepared monograph.

The book is presented as an old-fashioned treatise, the kind favoured by 19th century mathematicians and mechanicians, complete with a 26-word descriptive subtitle. It is clearly a labour of love. The author's devotion to the subject is conspicuous, although it is sullied somewhat by his characterization of competing ideas as merely "popular," "religiously promoted," "cosmetic," "fashionable." He avoids "modern applied mathematics," and "epsilonics," since "libraries are full of such rigorous/formalistic books that are almost never read." Although his book is evidently not "rigorous," it nonetheless requires 23 pages of "Summary of Conventions, Notations, and Basic Formulae." As an example, this Summary begins by explaining that "Chapters are divided into sections; e.g., Section 3.4 means Section 4 of Chapter 3." The numbering scheme for equations, examples, and problems is described in even more detail. An additional 250+ footnotes provide further clarification as required beyond the prefatory material. For example, the symbol $\mathbf{S}(\ldots)$ representing material summation, used extensively in Chapter 6, is not defined in the prefix, but rather in a brief footnote on p. 240, where the reader learns that $\mathbf{S}(\ldots)$ "may [be replaced] with the more familiar Leibnizian $\int(\ldots)$."

The author's opinion on various matters is perhaps too clear. In the acknowledgments, the author identifies those who oppose the institution of tenure as "reactionary ideologues, demagogues, and ignoramuses." Maybe such inflammatory language is appropriate for commentary in the introduction material of a technical monograph; however, this approach is applied equally to technical issues of the text. For example, "the currently popular and religiously promoted matrix notation" is recognized as a "side effect of single-minded and rabid… computeritis." "Order matters" in the tensor product $\boldsymbol{e}^1 \otimes \boldsymbol{e}^k$ (p. xxv), but the fact that order matters in matrix multiplication is billed as a "merciless straightjacket" (p. xii).

On the controversy* surrounding "Kane's Equations," Papastavridis settles firmly in the Appellian camp, characterizing Kane's approach as "arcane" and reflective of "a primitive and incomplete understanding of the general principles and mathematical structure of the equations of dynamics," "at odds with all reputable expositions on the subject." Indeed, Kane's approach is declared to be "a conceptually impoverished, isolated, and unmotivated scheme that soon leads … to a dynamical dead end." Well, reasonable people may disagree about the uniqueness and the usefulness of Kane's equations, and perhaps only those interested in "computational efficiency" will discover their utility. Those who have been inoculated against "computeritis" will no doubt reject an approach long advertised as a means of improving the efficiency of dynamical simulation of complex systems such as aircraft, spacecraft, and ships.

The book ends rather abruptly, concluding with an example application involving the motion of a particle moving in a uniformly rotating frame of reference.

References

[1]Aris, R., *Vectors, Tensors, and the Basic Equations of Fluid Mechanics*, Prentice–Hall, Englewood Cliffs, NJ, 1962. Reprinted by Dover Publications, New York, 1990.

[2]DesLoge, E. A., "Relationship Between Kane's Equations and The Gibbs-Appell Equations," *Journal of Guidance, Control, and Dynamics*, Vol. 10, No. 1, 1987, pp. 120–122.

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*See Ref. 2 and subsequent Comments in Vols. 10 and 11 of the *Journal*.